AMENDMENTS TO THE CLAIMS

This listing of claims will replace all prior versions, and listings, of claims in the application:

Listing of Claims:

1. (Currently amended) A method for using a computer system to solve an unconstrained interval global optimization problem specified by a function *f*, wherein *f* is a scalar function of a vector $\mathbf{x} = (x_1, x_2, x_3, \ldots x_n)$, the method comprising:

receiving a representation of the function *f* at the computer system;

storing the representation in a memory within the computer system; and

performing an interval global optimization process to compute guaranteed bounds on a globally minimum value of the function $f(\mathbf{x})$ over a subbox **X**;

wherein performing the interval global optimization process involves,

applying term consistency to a set of relations associated with the function *f* over the subbox **X**, and excluding any portion of the subbox **X** that violates any of these relations,

applying box consistency to the set of relations associated with the function *f* over the subbox **X**, and excluding any portion of the subbox **X** that violates any of these relations, and

performing an interval Newton step on the subbox **X** to produce a resulting subbox **Y**, wherein the point of expansion of the interval Newton step is a point **x** within the subbox **X**, and wherein performing the interval Newton step involves evaluating the gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$ using interval arithmetic; and

recording the guaranteed bounds in the memory within the computer system.

2. (Original) The method of claim 1, wherein applying term consistency involves:

symbolically manipulating an equation within the computer system to solve for a term $g(x_j)$, thereby producing a modified equation $g(x_j) = h(\mathbf{x})$, wherein the term $g(x_j)$ can be analytically inverted to produce an inverse function $g^{-1}(y)$;

substituting the subbox $\mathbf{X}$ into the modified equation to produce the equation $g(X'_j) = h(\mathbf{X})$;

solving for $X'_j = g^{-1}(h(\mathbf{X}))$; and

intersecting X'_j with the interval X_j to produce a new subbox $\mathbf{X}^+$;

wherein the new subbox $\mathbf{X}^+$ contains all solutions of the equation within the subbox $\mathbf{X}$, and wherein the size of the new subbox $\mathbf{X}^+$ is less than or equal to the size of the subbox $\mathbf{X}$.

3. (Original) The method of claim 1, wherein performing the interval global optimization process involves:

keeping track of a smallest upper bound *f_bar* of the function $f(\mathbf{x})$;

removing from consideration any subbox $\mathbf{X}$ for which $f(\mathbf{X}) >$ *f_bar*; and

wherein applying term consistency to the *f_bar* relation involves applying term consistency to the inequality $f(\mathbf{x}) \leq$ *f_bar* over the subbox $\mathbf{X}$.

4. (Original) The method of claim 3, wherein applying box consistency to the set of relations involves applying box consistency to the inequality $f(\mathbf{x}) \leq$ *f_bar* over the subbox $\mathbf{X}$.

5. (Original) The method of claim 1, wherein performing the interval global optimization process involves:

determining the gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ includes components $g_i(\mathbf{x})$ (i=1, ..., n);

removing from consideration any subbox for which any element of $\mathbf{g}(\mathbf{x})$ is bounded away from zero, thereby indicating that the subbox does not include a stationary point of $f(\mathbf{x})$; and

wherein applying term consistency to the set of relations involves applying term consistency to each component $g_i(\mathbf{x})=0$ (i=1, ..., n) of $\mathbf{g}(\mathbf{x})=\mathbf{0}$ over the subbox $\mathbf{X}$.

6. (Original) The method of claim 5, wherein applying box consistency to the set of relations involves applying box consistency to each component $g_i(\mathbf{x})=0$ (i=1, ..., n) of $\mathbf{g}(\mathbf{x})=\mathbf{0}$ over the subbox $\mathbf{X}$.

7. (Original) The method of claim 1, wherein performing the interval global optimization process involves:

determining diagonal elements $H_{ii}(\mathbf{x})$ (i=1, ..., n) of the Hessian of the function $f(\mathbf{x})$;

removing from consideration any subbox for which a diagonal element of the Hessian is always negative, which indicates that the function f is not convex and consequently does not contain a global minimum within the subbox;

wherein applying term consistency to the set of relations involves applying term consistency to each inequality $H_{ii}(\mathbf{x}) \geq 0$ (i=1, ..., n) over the subbox $\mathbf{X}$.

8. (Original) The method of claim 7, wherein applying box consistency to the set of relations involves applying box consistency to each inequality $H_{ii}(\mathbf{x}) \geq 0$ (i=1, ..., n) over the subbox $\mathbf{X}$.

9. (Original) The method of claim 1,

wherein performing the interval Newton step involves,

computing the Jacobian $\mathbf{J}(\mathbf{x},\mathbf{X})$ of the gradient $\mathbf{g}$ evaluated as a function of a point $\mathbf{x}$ over the subbox $\mathbf{X}$,

computing an approximate inverse $\mathbf{B}$ of the center of $\mathbf{J}(\mathbf{x},\mathbf{X})$, and

using the approximate inverse $\mathbf{B}$ to analytically determine the system $\mathbf{Bg}(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ is the gradient of the function $f(\mathbf{x})$, and wherein $\mathbf{g}(\mathbf{x})$ includes components $g_i(\mathbf{x})$ $(i=1,\ldots,n)$; and

wherein applying term consistency to the set of relations involves applying term consistency to each component $(\mathbf{Bg}(\mathbf{x}))_i = 0$ $(i=1,\ldots,n)$ for each variable x_i $(i=1,\ldots,n)$ over the subbox $\mathbf{X}$.

10. (Original) The method of claim 9, wherein applying box consistency to the set of relations involves applying box consistency to each component $(\mathbf{Bg}(\mathbf{x}))_i = 0$ $(i=1,\ldots,n)$ for each variable x_i $(i=1,\ldots,n)$ over the subbox $\mathbf{X}$.

11. (Original) The method of claim 1, further comprising terminating attempts to further reduce the subbox $\mathbf{X}$ when:

the width of $\mathbf{X}$ is less than a first threshold value; and

the magnitude of $f(\mathbf{X})$ is less than a second threshold value.

12. (Original) The method of claim 11, wherein performing the interval Newton step involves:

computing $\mathbf{J}(\mathbf{x},\mathbf{X})$, wherein $\mathbf{J}(\mathbf{x},\mathbf{X})$ is the Jacobian of the function $\mathbf{f}$ evaluated as a function of $\mathbf{x}$ over the subbox $\mathbf{X}$; and

determining if $\mathbf{J}(\mathbf{x},\mathbf{X})$ is regular as a byproduct of solving for the subbox $\mathbf{Y}$ that contains values of $\mathbf{y}$ that satisfy $\mathbf{M}(\mathbf{x},\mathbf{X})(\mathbf{y}-\mathbf{x}) = \mathbf{r}(\mathbf{x})$, where

$\mathbf{M}(\mathbf{x},\mathbf{X}) = \mathbf{BJ}(\mathbf{x},\mathbf{X})$, $\mathbf{r}(\mathbf{x}) = -\mathbf{Bf}(\mathbf{x})$, and $\mathbf{B}$ is an approximate inverse of the center of $\mathbf{J}(\mathbf{x},\mathbf{X})$.

13. (Original) A computer-readable storage medium storing instructions that when executed by a computer cause the computer to perform a method for using a computer system to solve an unconstrained interval global optimization problem specified by a function f, wherein f is a scalar function of a vector $\mathbf{x} = (x_1, x_2, x_3, \ldots x_n)$, the method comprising:

receiving a representation of the function f at the computer system;

storing the representation in a memory within the computer system; and

performing an interval global optimization process to compute guaranteed bounds on a globally minimum value of the function $f(\mathbf{x})$ over a subbox $\mathbf{X}$;

wherein performing the interval global optimization process involves,

applying term consistency to a set of relations associated with the function f over the subbox $\mathbf{X}$, and excluding any portion of the subbox $\mathbf{X}$ that violates any of these relations,

applying box consistency to the set of relations associated with the function f over the subbox $\mathbf{X}$, and excluding any portion of the subbox $\mathbf{X}$ that violates any of these relations, and

performing an interval Newton step on the subbox $\mathbf{X}$ to produce a resulting subbox $\mathbf{Y}$, wherein the point of expansion of the interval Newton step is a point $\mathbf{x}$ within the subbox $\mathbf{X}$, and wherein performing the interval Newton step involves evaluating the gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$ using interval arithmetic.

14. (Original) The computer-readable storage medium of claim 13, wherein applying term consistency involves:

symbolically manipulating an equation within the computer system to solve for a term $g(x_j)$, thereby producing a modified equation $g(x_j) = h(\mathbf{x})$, wherein the term $g(x_j)$ can be analytically inverted to produce an inverse function $g^{-1}(y)$;

substituting the subbox $\mathbf{X}$ into the modified equation to produce the equation $g(X'_j) = h(\mathbf{X})$;

solving for $X'_j = g^{-1}(h(\mathbf{X}))$; and

intersecting X'_j with the interval X_j to produce a new subbox $\mathbf{X}^+$;

wherein the new subbox $\mathbf{X}^+$ contains all solutions of the equation within the subbox $\mathbf{X}$, and wherein the size of the new subbox $\mathbf{X}^+$ is less than or equal to the size of the subbox $\mathbf{X}$.

15. (Original) The computer-readable storage medium of claim 13, wherein performing the interval global optimization process involves:

keeping track of a smallest upper bound *f_bar* of the function $f(\mathbf{x})$;

removing from consideration any subbox $\mathbf{X}$ for which $f(\mathbf{X}) > f\_bar$; and

wherein applying term consistency to the *f_bar* relation involves applying term consistency to the inequality $f(\mathbf{x}) \leq f\_bar$ over the subbox $\mathbf{X}$.

16. (Original) The computer-readable storage medium of claim 15, wherein applying box consistency to the set of relations involves applying box consistency to the inequality $f(\mathbf{x}) \leq f\_bar$ over the subbox $\mathbf{X}$.

17. (Original) The computer-readable storage medium of claim 13, wherein performing the interval global optimization process involves:

determining the gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ includes components $g_i(\mathbf{x})$ $(i=1, \ldots, n)$;

removing from consideration any subbox for which any element of $\mathbf{g}(\mathbf{x})$ is bounded away from zero, thereby indicating that the subbox does not include a stationary point of $f(\mathbf{x})$; and

wherein applying term consistency to the set of relations involves applying term consistency to each component $g_i(\mathbf{x})=0$ $(i=1,\ldots,n)$ of $\mathbf{g}(\mathbf{x})=\mathbf{0}$ over the subbox $\mathbf{X}$.

18. (Original) The computer-readable storage medium of claim 17, wherein applying box consistency to the set of relations involves applying box consistency to each component $g_i(\mathbf{x})=0$ $(i=1,\ldots,n)$ of $\mathbf{g}(\mathbf{x})=\mathbf{0}$ over the subbox $\mathbf{X}$.

19. (Original) The computer-readable storage medium of claim 13, wherein performing the interval global optimization process involves:

determining diagonal elements $H_{ii}(\mathbf{x})$ $(i=1,\ldots,n)$ of the Hessian of the function $f(\mathbf{x})$;

removing from consideration any subbox for which a diagonal element of the Hessian is always negative, which indicates that the function f is not convex and consequently does not contain a global minimum within the subbox;

wherein applying term consistency to the set of relations involves applying term consistency to each inequality $H_{ii}(\mathbf{x}) \geq 0$ $(i=1,\ldots,n)$ over the subbox $\mathbf{X}$.

20. (Original) The computer-readable storage medium of claim 19, wherein applying box consistency to the set of relations involves applying box consistency to each inequality $H_{ii}(\mathbf{x}) \geq 0$ $(i=1,\ldots,n)$ over the subbox $\mathbf{X}$.

21. (Original) The computer-readable storage medium of claim 13,

wherein performing the interval Newton step involves,

computing the Jacobian $\mathbf{J}(\mathbf{x},\mathbf{X})$ of the gradient $\mathbf{g}$ evaluated as a function of a point $\mathbf{x}$ over the subbox $\mathbf{X}$,

computing an approximate inverse $\mathbf{B}$ of the center of $\mathbf{J}(\mathbf{x},\mathbf{X})$, and

using the approximate inverse $\mathbf{B}$ to analytically determine the system $\mathbf{Bg}(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ is the gradient of the function $f(\mathbf{x})$, and wherein $\mathbf{g}(\mathbf{x})$ includes components $g_i(\mathbf{x})$ $(i=1,\ldots,n)$; and

wherein applying term consistency to the set of relations involves applying term consistency to each component $(\mathbf{Bg}(\mathbf{x}))_i = 0$ $(i=1,\ldots,n)$ for each variable x_i $(i=1,\ldots,n)$ over the subbox $\mathbf{X}$.

22. (Original) The computer-readable storage medium of claim 21, wherein applying box consistency to the set of relations involves applying box consistency to each component $(\mathbf{Bg}(\mathbf{x}))_i = 0$ $(i=1,\ldots,n)$ for each variable x_i $(i=1,\ldots,n)$ over the subbox $\mathbf{X}$.

23. (Original) The computer-readable storage medium of claim 13, wherein the method further comprises terminating attempts to further reduce the subbox $\mathbf{X}$ when:

the width of $\mathbf{X}$ is less than a first threshold value; and

the magnitude of $f(\mathbf{X})$ is less than a second threshold value.

24. (Original) The computer-readable storage medium of claim 13, wherein performing the interval Newton step involves:

computing $\mathbf{J}(\mathbf{x},\mathbf{X})$, wherein $\mathbf{J}(\mathbf{x},\mathbf{X})$ is the Jacobian of the function $\mathbf{f}$ evaluated as a function of $\mathbf{x}$ over the subbox $\mathbf{X}$; and

determining if $\mathbf{J}(\mathbf{x},\mathbf{X})$ is regular as a byproduct of solving for the subbox $\mathbf{Y}$ that contains values of $\mathbf{y}$ that satisfy $\mathbf{M}(\mathbf{x},\mathbf{X})(\mathbf{y}-\mathbf{x}) = \mathbf{r}(\mathbf{x})$, where

$\mathbf{M}(\mathbf{x},\mathbf{X}) = \mathbf{BJ}(\mathbf{x},\mathbf{X})$, $\mathbf{r}(\mathbf{x}) = -\mathbf{Bf}(\mathbf{x})$, and $\mathbf{B}$ is an approximate inverse of the center of $\mathbf{J}(\mathbf{x},\mathbf{X})$.

25. (Currently amended) An apparatus that solves an unconstrained interval global optimization problem specified by a function *f*, wherein *f* is a scalar function of a vector $\mathbf{x} = (x_1, x_2, x_3, \dots x_n)$, the apparatus comprising:

a receiving mechanism that is configured to receive a representation of the function *f*;

a memory for storing the representation; and

an interval global optimization mechanism that is configured to perform an interval global optimization process to compute guaranteed bounds on a globally minimum value of the function $f(\mathbf{x})$ over a subbox $\mathbf{X}$;

a term consistency mechanism within the interval global optimization mechanism that is configured to apply term consistency to a set of relations associated with the function *f* over the subbox $\mathbf{X}$, and to exclude any portion of the subbox $\mathbf{X}$ that violates any of these relations;

a box consistency mechanism within the interval global optimization mechanism that is configured to apply box consistency to the set of relations associated with the function *f* over the subbox $\mathbf{X}$, and to exclude any portion of the subbox $\mathbf{X}$ that violates any of these relations; ~~and~~

an interval Newton mechanism within the interval global optimization mechanism that is configured to perform an interval Newton step on the subbox $\mathbf{X}$ to produce a resulting subbox $\mathbf{Y}$, wherein the point of expansion of the interval Newton step is a point $\mathbf{x}$ within the subbox $\mathbf{X}$, and wherein performing the interval Newton step involves evaluating the gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$ using interval arithmetic; and

a recording mechanism configured to record the guaranteed bounds in the memory.

26. (Original) The apparatus of claim 25, wherein the term consistency mechanism is configured to:

symbolically manipulate an equation to solve for a term $g(x_j)$, thereby producing a modified equation $g(x_j) = h(\mathbf{x})$, wherein the term $g(x_j)$ can be analytically inverted to produce an inverse function $g^{-1}(y)$;

substitute the subbox $\mathbf{X}$ into the modified equation to produce the equation $g(X'_j) = h(\mathbf{X})$;

solve for $X'_j = g^{-1}(h(\mathbf{X}))$; and to

intersect X'_j with the interval X_j to produce a new subbox $\mathbf{X}^+$;

wherein the new subbox $\mathbf{X}^+$ contains all solutions of the equation within the subbox $\mathbf{X}$, and wherein the size of the new subbox $\mathbf{X}^+$ is less than or equal to the size of the subbox $\mathbf{X}$.

27. (Original) The apparatus of claim 25,

wherein the interval global optimization mechanism is configured to,

keep track of a smallest upper bound *f_bar* of the function $f(\mathbf{x})$, and to

remove from consideration any subbox $\mathbf{X}$ for which $f(\mathbf{X}) > f\_bar$; and

wherein the term consistency mechanism is configured to apply term consistency to the inequality $f(\mathbf{x}) \leq f\_bar$ over the subbox $\mathbf{X}$.

28. (Original) The apparatus of claim 27, wherein the box consistency mechanism is configured to apply box consistency to the inequality $f(\mathbf{x}) \leq f\_bar$ over the subbox $\mathbf{X}$.

29. (Original) The apparatus of claim 25,

wherein the interval global optimization mechanism is configured to,

determine the gradient $\mathbf{g}(\mathbf{x})$ of the function $f(\mathbf{x})$, wherein $\mathbf{g}(\mathbf{x})$ includes components $g_i(\mathbf{x})$ $(i=1,\ldots,n)$, and to

remove from consideration any subbox for which any element of $\mathbf{g}(\mathbf{x})$ is bounded away from zero, thereby indicating that the subbox does not include a stationary point of $f(\mathbf{x})$; and

wherein the term consistency mechanism is configured to apply term consistency to each component $g_i(\mathbf{x})=0$ $(i=1,\ldots,n)$ of $\mathbf{g}(\mathbf{x})=\mathbf{0}$ over the subbox $\mathbf{X}$.

30. (Original) The apparatus of claim 29, wherein the box consistency mechanism is configured to apply box consistency to each component $g_i(\mathbf{x})=0$ $(i=1,\ldots,n)$ of $\mathbf{g}(\mathbf{x})=\mathbf{0}$ over the subbox $\mathbf{X}$.

31. (Original) The apparatus of claim 25,

wherein the interval global optimization mechanism is configured to,

determine diagonal elements $H_{ii}(\mathbf{x})$ $(i=1,\ldots,n)$ of the Hessian of the function $f(\mathbf{x})$, and to

remove from consideration any subbox for which a diagonal element of the Hessian is always negative, which indicates that the function f is not convex and consequently does not contain a global minimum within the subbox;

wherein the term consistency mechanism is configured to apply term consistency to each inequality $H_{ii}(\mathbf{x}) \geq 0$ $(i=1,\ldots,n)$ over the subbox $\mathbf{X}$.

32. (Original) The apparatus of claim 31, wherein the box consistency mechanism is configured to apply box consistency to each inequality $H_{ii}(\mathbf{x}) \geq 0$ $(i=1,\ldots,n)$ over the subbox $\mathbf{X}$.

33. (Original) The apparatus of claim 25,

wherein the interval Newton mechanism is configured to,

compute the Jacobian **J**(**x**,**X**) of the gradient **g** evaluated as a function of a point **x** over the subbox **X**,

compute an approximate inverse **B** of the center of **J**(**x**,**X**), and to

use the approximate inverse **B** to analytically determine the system **Bg**(**x**), wherein **g**(**x**) is the gradient of the function f(**x**), and wherein **g**(**x**) includes components g_i(**x**) ($i=1,\ldots,n$); and

wherein the term consistency mechanism is configured to apply term consistency to each component $(\mathbf{Bg}(\mathbf{x}))_i = 0$ ($i=1,\ldots,n$) for each variable x_i ($i=1,\ldots,n$) over the subbox **X**.

34. (Original) The apparatus of claim 33, wherein the box consistency mechanism is configured to apply box consistency to each component $(\mathbf{Bg}(\mathbf{x}))_i = 0$ ($i=1,\ldots,n$) for each variable x_i ($i=1,\ldots,n$) over the subbox **X**.

35. (Original) The apparatus of claim 25, further comprising a termination mechanism that is configured to terminate attempts to further reduce the subbox **X** when:

the width of **X** is less than a first threshold value; and

the magnitude of f(**X**) is less than a second threshold value.

36. (Currently amended) The apparatus of ~~claim 11~~claim 35, wherein the interval Newton mechanism is configured to:,

compute **J**(**x**,**X**), wherein **J**(**x**,**X**) is the Jacobian of the function **f** evaluated as a function of **x** over the subbox **X**; and to

determine if **J**(**x**,**X**) is regular as a byproduct of solving for the subbox **Y** that contains values of **y** that satisfy **M**(**x**,**X**)(**y**-**x**) = **r**(**x**), where

$\mathbf{M}(\mathbf{x},\mathbf{X}) = \mathbf{BJ}(\mathbf{x},\mathbf{X})$, $\mathbf{r}(\mathbf{x}) = -\mathbf{Bf}(\mathbf{x})$, and $\mathbf{B}$ is an approximate inverse of the center of $\mathbf{J}(\mathbf{x},\mathbf{X})$.